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Live sound subwoofer system performance quantification

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ABSTRACT

The general aim of live sound reinforcement is to deliver an appropriate and consistent listening experience across an audience. Achieving this in the subwoofer range (typically between 20 – 100 Hz) has been the focus of previous work, where techniques have been developed to allow for consistent sound energy distribution over a wide area. While this provides system designers with a powerful set of tools, it brings with it many potential metrics to quantify performance. This research identifies key indicators of subwoofer system performance and proposes a single weighted metric to quantify overall performance. Both centrally-distributed and left/right configurations are analyzed using the new metric to highlight functionality.

1 Introduction

A recent theme within the live sound reinforcement community has been the idea of the “democracy of sound” [1]. In principle, all members of an audience should receive the same audio content, regardless of location (within reason). While it is unreasonable to expect consistent sound levels across an entire venue (due to propagation loss) – at least without a complicated system – it isn’t out of the question to expect consistent tonality at all audience locations.

Over the past few decades, the use of line array technology has significantly improved the consistency of tonality in the non-subwoofer range (above roughly 100 Hz) due to accurately controlled horizontal and vertical dispersion. Optimization techniques are largely mature now and are well-known to system engineers [2,3].

Over the subwoofer range (20 – 100 Hz) there also exists a wealth of knowledge on optimization (mostly focused on achieving the desired coverage while limiting sound energy outside an audience area) [4,5,6,7,8]. Typically, a system engineer will space individual subwoofers according to the half-wavelength of a frequency at (or near) the upper limit of the subwoofer range (to allow for source-to-source coupling) and will apply time delay to individual subwoofers to widen/narrow the coverage pattern. Some engineers also apply amplitude tapering to reduce lobing, although others avoid this since it potentially reduces overall system output [1].

Compromises are required when optimizing a system. First, the number of subwoofers available is limited by company stock, truck space, amplifier/processing channels, power distribution, etc. [6]. On site, there will be a limited area which can accommodate loudspeakers. Rarely will a system be able to protrude significantly in front of or to the side of a stage. Sometimes there is a central walkway into the audience, thus preventing placement of a centrally-distributed system.

Once subwoofers are physically placed and patched, further compromises are required. There is generally a trade-off between coverage width and front-of-house (FOH, a.k.a. mix position) sound level. The system must be optimized so that the FOH engineer as well as the audience receive acceptable levels (not to mention consistent tonality).

The final challenge in system optimization is time. There is usually limited time on site for fine-tuning.

Ideally, virtual optimization is carried out using manufacturer-provided software [9,10,11] to save time on site. Most engineers use such software, analyzing sound pressure level (SPL) distribution plots at discrete frequencies and phase responses in relation to other sub-systems. While such software has proven indispensable in sound system design, the process requires an amount of trial-and-error to meet all requirements as best as possible.

This research sets out to derive a single performance metric for subwoofer systems that can be used by system engineers to automatically identify ideal calibration settings using a set of constraints. The aim is to allow for an agreeable compromise to be found between consistent tonality, consistent audience sound level and acceptable system headroom. Detailed control of system directionality (in terms of SPL on stage or in other sound-sensitive areas) isn't addressed in this work since it has been the focus of previous work [6,8,12], but could be incorporated into the proposed performance metric as part of further development.

2 Quantification of performance

Three central objectives can be identified in relation to subwoofer system performance at live events:

1) Tonal consistency across an audience
2) Acceptable system headroom
3) Minimal difference between the mix position and mean audience level

There are two customers (so to speak): the audience and the FOH engineer. Both should be considered since the FOH engineer must receive an acceptable sound level as well as receive an accurate representation of what the audience is hearing.

The following sections detail the process of quantifying these three individual performance metrics and how they can then be used to formulate a single indicator of subwoofer system performance.

2.1 Tonal consistency

Tonal consistency across an audience can be quantified by mean spatial variance (SV) which calculates the average variance in magnitude response across a set of measurement points, given in decibels (Eq. 2.1) [13]. Note that an alternative method of calculating SV exists, whereby the calculation is performed using average standard deviation rather than variance [14]. Either method is acceptable and shows identical trends. This work uses the variance method exclusively.

$$SV = \frac{1}{N_f}\sum_{i=f_{lo}}^{f_{hi}}\frac{1}{N_p - 1}\sum_{p=1}^{N_p}\left(L_p(p,i) - \overline{L_p(i)}\right)^2 \quad (2.1)$$

where, spatial variance (SV, in dB) is calculated based on the number of frequency bins analyzed (N_f), the frequency range (f_{lo} to f_{hi}) using linearly-spaced frequency bins, the number of measurement points (N_p), the sound pressure level at point p and frequency i ($L_p(p,i)$) and the mean sound pressure level across all measurement points at frequency i ($\overline{L_p(i)}$). SV ranges from 0 dB (no variation across the audience) upwards.

Since audience members are usually spread over a wide area, propagation loss is a factor. The SV calculation is blind to this, therefore all magnitude responses must be normalized over the subwoofer range so that SV measures tonal consistency rather than propagation loss. In this work, MATLAB's *msnorm* function was used for normalization [15].

2.2 Acceptable system headroom

Change in system headroom is found using Eq. 2.2:

$$\Delta HR = L_{p,tar} - L_{p,foh} \quad (2.2)$$

where ΔHR (dB) is the difference between target FOH SPL, $L_{p,tar}$ (dB), and acheived FOH SPL, $L_{p,foh}$ (dB). If ΔHR is less than 0 dB (indicating SPL at the mix position exceeds the target), then ΔHR is fixed at 0 dB since the target has been met. If the original system headroom minus ΔHR is below 6 dB, then ΔHR is fixed at ∞ dB, as this represents insufficient headroom to operate the system. The original system headroom is calculated based on the initial subwoofer output at 1 m and the maximum possible subwoofer output at 1 m (set to 120 dB and 140 dB [16], respectively, for all examples here).

A difference between target and achieved FOH SPL is typically addressed by boosting output to the subwoofer system, so ΔHR is a good indicator of system headroom. A smaller ΔHR means more headroom available within the system.

2.3 Audience and FOH level consistency

Unoptimized subwoofer systems suffer from what's referred to as "power alley". This is where subwoofer outputs constructively sum in the central region of an audience, resulting in high SPL. In cases such as this, the FOH engineer may receive significantly more low-frequency energy than much of the audience, which can result in a bass-light mix in any non-central listening area [6].

With this in mind, it's essential to ensure an optimized subwoofer system achieves consistent SPL between FOH and the audience. While it's unreasonable to expect perfectly consistent SPL across the entire audience (due to propagation loss – especially for ground-based systems [6]), achieving similar FOH SPL and mean audience SPL is within reason. The difference between FOH SPL and mean audience SPL can be calculated using Eq. 2.3:

$$\Delta AUD = \left| L_{p,FOH} - MOL \right| \tag{2.3}$$

where ΔAUD (dB) is determined by the absolute value of the difference between the SPL at FOH, $L_{p,foh}$ (dB), and the mean output level (MOL, dB) across the audience. Mean output level is found with Eq. 2.4 [13]:

$$MOL = \frac{1}{N_f N_p} \sum_{i=f_{lo}}^{f_{hi}} \sum_{p=1}^{N_p} L_p(p,i) \tag{2.4}$$

2.4 Array performance rating (APR)

The individual metrics described in Sections 2.1 – 2.3 can be formulated into a single performance indicator to give an overall system rating (Eq. 2.5).

$$APR = W_{SV}\left(10^{\frac{-SV}{20}}\right) + W_{HR}\left(10^{\frac{-\Delta HR}{20}}\right) + W_{AUD}\left(10^{\frac{-\Delta AUD}{20}}\right) \tag{2.5}$$

Users are able to prioritize each of the three metrics as they see fit using weighting. The weighting values W_{SV}, W_{HR} and W_{AUD} correspond to the individual metrics SV, ΔHR and ΔAUD, respectively. Each weighting can take on a value from 0 (not considered) to 1 (exclusively considered), so long as all three weightings sum to 1. In this work all metrics are given an equal weighting of 1/3.

Each individual metric is converted to linear scale. Since all have an ideal value of 0 dB, the linear range spans 0 (worst) to 1 (best). To clarify, APR can be mapped to letter grades as given in Table 2.1:

Grade	*APR range*
A	[0.80 – 1.00]
B	[0.65 – 0.80)
C	[0.50 – 0.65)
D	[0.35 – 0.50)
F	[0.00 – 0.35)

Table 2.1 APR letter grading scale

3 Individual unit efficiency

Before looking into subwoofer system performance, it is instructive to inspect individual units and clusters (in this work clusters refer to two or more individual subwoofers positioned in close proximity to achieve a desired polar response). It is common for engineers to use subwoofer clusters to steer sound energy towards the audience and away from the stage and other noise-sensitive areas.

Since this work focuses only on array optimization (not on loudspeaker design), subwoofers were modeled as point sources to avoid an overly-complicated simulation. A single source was centered 1 m in front of the stage (20 m x 8 m) and FOH (4 m x 4 m) was positioned 20 m from the front of the stage. The audience area began one meter in front of the subwoofer, with measurement points located every 1.715 m, corresponding to the half-wavelength of 100 Hz (the upper limit of the subwoofer band). Henceforth, any reference to source spacing will be in terms of frequency (representing the upper source coupling limit). The overall audience spans an area of 40 m x 50 m and consists of 775 points (Fig. 3.1).

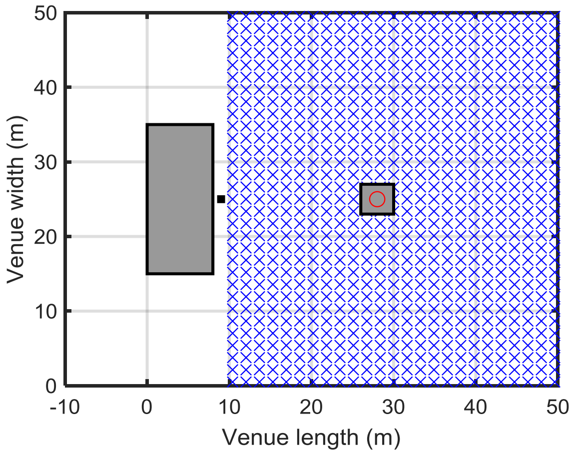

Fig. 3.1 Layout for a single omni subwoofer (■ = subwoofers, x = measurement points)

In order to achieve directionality, a second subwoofer is required, where a cardioid response is achieved using either a gradient [17] or end-fire [18] configuration. In general, the gradient configuration gives better low-frequency rejection on stage with slightly smeared transients in the audience, while the end-fire configuration gives less rejection on stage, but does not degrade the transient response [19].

In both cases, the second subwoofer was placed directly behind the primary subwoofer by 85 cm (1/4 wavelength at 100 Hz). The omnidirectional, gradient and end-fire configurations were simulated, collecting the metrics discussed in Section 2, as shown in Table 3.1. Target FOH SPL was set at 115 dB (as with all examples given in this work).

Metric	*Omni*	*Gradient*	*End-fire*
APR	0.60	0.58	0.60
SV	0.00	0.07	0.03
ΔHR	∞	∞	∞
ΔAUD	1.84	2.65	1.90

Table 3.1 Metrics for the single unit and cluster tests (all values other than APR are given in dB)

The most serious issue is seen in the ΔHR values. This represents how well (or not) a system achieves the required FOH SPL. Each configuration runs out of headroom before achieving the target FOH SPL, indicating a single source solution is unreasonable.

What is interesting here is that regardless of approach, APR is largely configuration-independent (the only difference being due to interference from the secondary units in the gradient configuration). This is useful, as it highlights that an array can be optimized in software using omnidirectional sources and then implemented with clusters (or directional sources) without significantly affecting APR.

4 Array efficiency

Since individual unit/cluster directionality has little effect on array optimization, the analysis can proceed using omnidirectional sources.

Two varieties of configurations are inspected: centrally-distributed and conventional left/right systems. In both cases, each individual source was set to output 120 dB at 1 m to approximate a system with ample headroom (20 dB) for peak handling.

4.1 Left/right configurations

Historically, live sound reinforcement systems have been set up in a left/right configuration [6]. Today this holds true for many subwoofer systems either using flown subwoofers alongside main arrays or using ground based systems (split into left/right arrays to allow central walkways into the audience).

Only ground-based systems are explored in this work because flown systems are typically limited to no more than two hangs per side of the stage, whereas ground-based systems can have a horizontally distributed array, space permitting.

Left/right configurations were analyzed based on the number of individual sources and source spacing. The number of sources in the array ranged from 2 to 30 subwoofers, while the source spacing ranged from 20 – 150 Hz. The number of sources was tested in increments of two and the source spacing was tested in increments of 10 Hz. During the first round of testing, the only limitation imposed was that the innermost left- and right-side subwoofers must be spaced by 15 m. APR was calculated for each test configuration (Fig. 4.1).

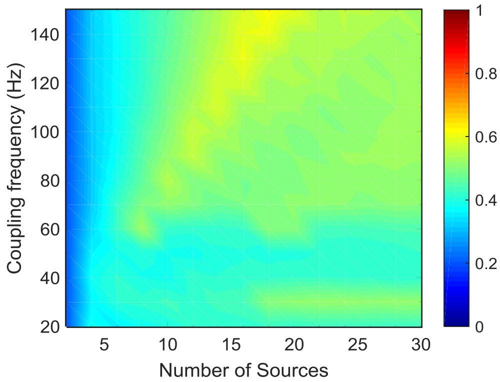

Fig. 4.1 APR for all investigated left/right subwoofer array configurations

The data shows that left/right configurations give moderate performance in terms of APR. The best performing configuration consists of 18 subwoofers (9 per side) with coupling up to 150 Hz (1.143 m spacing), giving an APR of 0.62 (C grade).

Unfortunately, this analysis overlooks the width of the system. If 30 units (15 subwoofers per side) were spaced at 20 Hz (8.575 m), the system would extend over 115 m from each side of the stage! Even the optimal 18-subwoofer configuration would require an array that extends 6.7 m beyond the side of the stage. This isn't likely to be practical.

The test, therefore, must be reconsidered with an imposed limitation on system width. In this case, subwoofer placement was only allowed within 3 m of each side of the stage (Fig. 4.2).

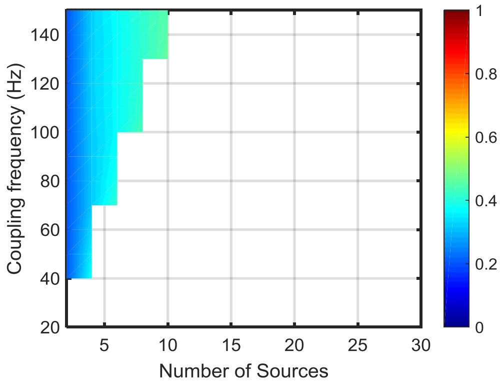

Fig. 4.2 APR for all investigated left/right subwoofer array configurations
(arrays must end within 3 m of each stage edge)

With the width limitation in place, the possible arrangements for a conventional left/right subwoofer array are severely limited. The configurations with only a few units per side don't provide adequate FOH SPL and suffer from problematic comb-filtering, thus limiting APR.

A system consisting of 10 subwoofers (5 per side) spaced at 130 Hz performs best with an APR of 0.46 (D grade). The normalized magnitude responses and SPL distribution for this system can be inspected to judge system performance (Figs. 4.3 and 4.4). All magnitude responses in this work have 1/9th octave smoothing, according to [20].

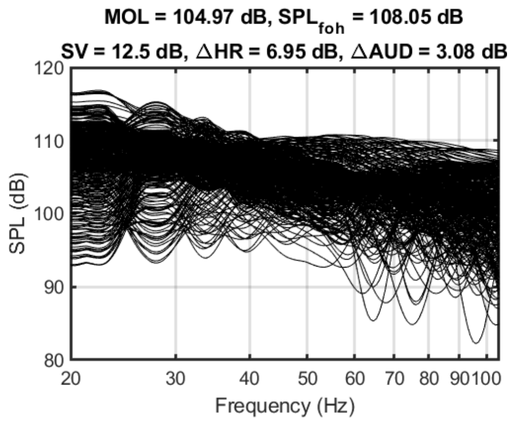

Fig. 4.3 Normalized magnitude responses for a 10-subwoofer left/right system with 130 Hz spacing

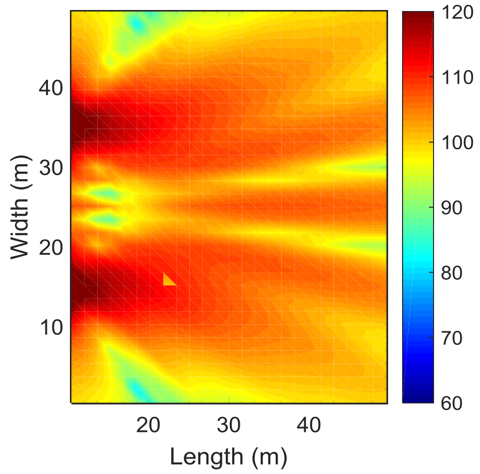

Fig. 4.4 SPL distribution (at 65 Hz) for a 10-subwoofer left/right system with 130 Hz spacing

FOH SPL is 108.05 dB and the MOL across the audience is 104.97 dB (fairly good agreement). Unfortunately, the system fails to deliver in terms of consistent tonality, registering an SV of 12.50 dB (a good system should have < 3dB SV).

While the SPL distribution at 65 Hz looks reasonable for a left/right configuration, the magnitude responses in Fig. 4.3 highlight severe fluctuations over the entire subwoofer range. As is, this isn't an ideal subwoofer system to use in practice; consistent tonality will not be achieved.

Since most problems here stem from coherent interference between the left and right sides of the system, any further optimization would be minimally effective, so won't be investigated here.

If a left/right system is unavoidable (due to such restrictions as mentioned earlier) it is recommended that some form of decorrelation is applied between the left and right components (or each subwoofer, if possible). Such techniques are known and have shown to be moderately effective in reducing severe comb-filtering within such systems [17,18,20].

4.2 Centrally-distributed configurations

Many system engineers employ centrally-distributed subwoofer arrays instead of left/right configurations. Central systems provide some advantages over left/right systems, the most important being that (if deployed properly) the system will behave as an array as opposed to a set of discrete sources. This allows for effective optimization, opening the possibility to exceptional system performance.

To investigate central array performance, an identical setup was simulated as in Section 4.1, but now with a central array as opposed to a left/right system. The same test variables were investigated with results after imposing a width restriction to within 3 m of the stage edges are shown in Fig. 4.5.

The best rated physical layout uses 22 subwoofers spaced at 140 Hz, giving an APR of 0.75 (B grade, already a significant improvement from the left/right system APR of 0.46, D grade). A 22-subwoofer system, however, isn't necessarily realistic (at least for all but the largest events). Restricting the number of subwoofers to 10 (which allows for a direct comparison to the best-case left/right configuration from Section 4.1) points to ideal source spacing at 60 Hz, giving an APR of 0.57 (C-grade). This represents a 0.11 improvement in APR, as compared to the best-case left/right configuration. The normalized magnitude responses of the central subwoofer array layout are given in Fig. 4.6.

The centrally-distributed array is about 8 dB below the desired FOH SPL of 107.20 dB, but with moderate agreement between FOH and audience levels (a 5.62 dB difference). The clear benefit, though, is with spatial variance. The central system gives 2.03 dB spatial variance across the audience, a marked improvement from 12.50 dB SV for the left/right system.

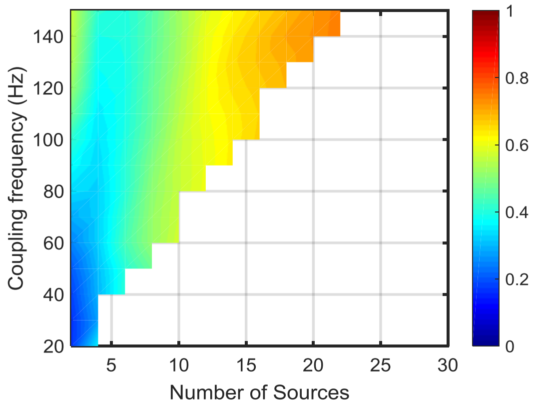

Fig. 4.5 APR for all investigated center subwoofer array configurations
(arrays must end within 3 m of each stage edge)

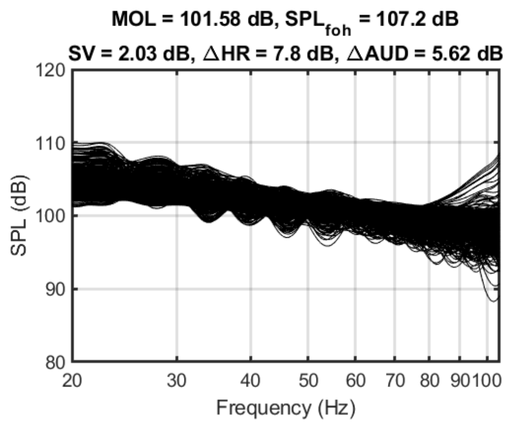

Fig. 4.6 Normalized magnitude responses for a 10-subwoofer central system with 60 Hz spacing

From here, further optimization can be pursued. First, individual subwoofer time delay is investigated. Calculating delay precisely to one point in the audience has been previously shown as ineffective [19]; therefore, delays are determined from a measurement point behind the array. This method creates a virtual point source behind the subwoofer array, whereby the width of the resulting coverage pattern is a function of the point's distance from the array [8]. This gives a more even delay arc as compared to perfectly delaying to a single point as highlighted in Figs. 4.7 and 4.8, respectively (alignment points at (30 m, 10 m) and (-8 m, 25 m) were used, respectively, for illustrative purposes).

The question is to which point should the array be aligned? To determine this, a series of simulations can be performed where the delay point is swept between potential distances from the array (Fig. 4.9).

The best possible delay point is located at (-17 m, 25 m), 26 m behind the array, resulting in an APR of 0.68 (B grade, an improvement of 0.11). Importantly, it can be seen that there is little change in performance between aligning at 20 m or 50 m behind the array (APR variability of only 0.03), indicating that precise alignment point location isn't critical. The normalized magnitude responses are given in Fig. 4.10.

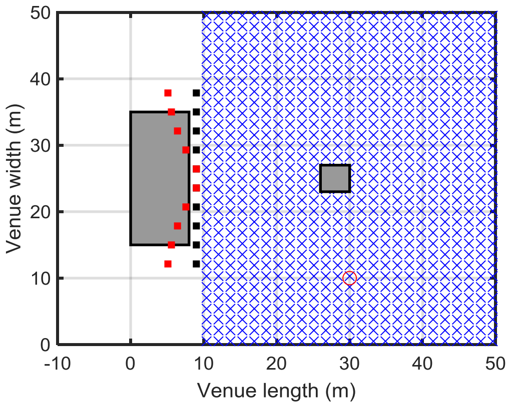

Fig. 4.7 Layout for the 22-unit central array delayed precisely to the delay point (■ = subwoofers, ■ = effective position after delay, x = measurement points, ○ = alignment point)

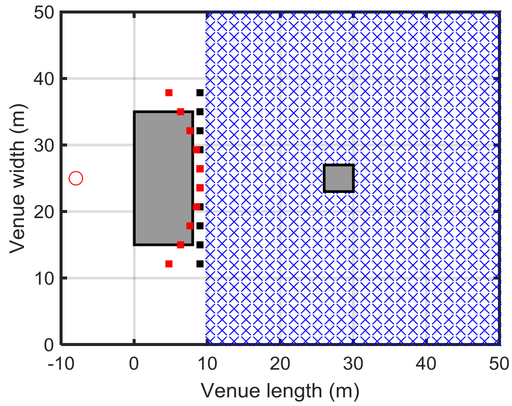

Fig. 4.8 Layout for the 22-unit central array delayed around the delay point (■ = subwoofers, ■ = effective position after delay, x = measurement points, ○ = alignment point)

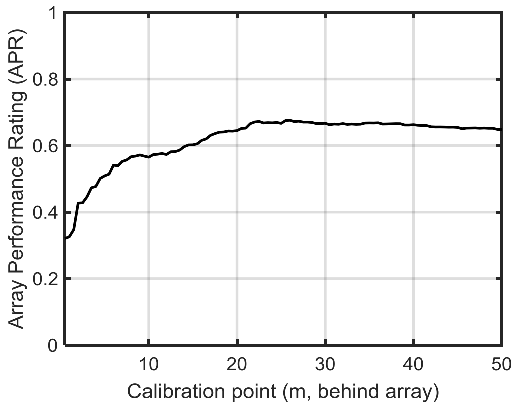

Fig. 4.9 APR after subwoofer delay applied over a range of positions behind the array

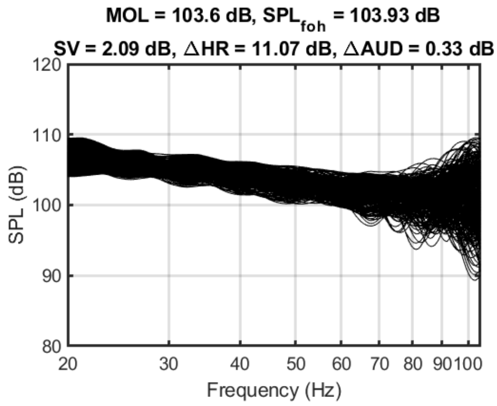

Fig. 4.10 Normalized magnitude responses for the 10-subwoofer system with subwoofer delay applied

The normalized magnitude responses highlight the improved system performance. Spatial variance, is about the same as before, at 2.09 dB. More importantly, the difference between FOH SPL and audience MOL has dropped from 5.62 dB to 0.33 dB – a significant improvement which means the FOH engineer will receive the same SPL as the audience.

As can be observed from the magnitude responses in Fig. 4.10, there is significant variance above 60 Hz (the array's upper coupling frequency limit). The effect is more pronounced after subwoofer delay has been applied. This issue can be addressed via so-called position compensation [19]. Delaying individual subwoofers increases the effective spacing between each unit. To address this, the effective spacing of the subwoofers is calculated and each unit's position is adjusted until all inter-unit spacings are equal. This raises APR to 0.70 (Fig. 4.11). In this scenario, position compensation has improved each of the three metrics.

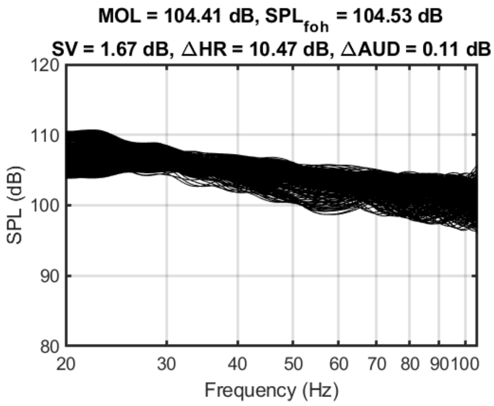

Fig. 4.11 Normalized magnitude responses for the 10-subwoofer system with subwoofer delay and position compensation applied

An additional optimization technique that some system engineers may choose to apply is amplitude tapering. This is where the output amplitude of each unit within the array is progressively attenuated when moving from the center to the outside of the array. In this work, tapering was applied using a Tukey window, according to [8], where the linear source gain vector was calculated to cover 4 additional subwoofers than are in the array. This avoids having any subwoofer in the array outputting negligible sound energy (the first and last two values in the vector aren't used). Note there are alternative methods for array amplitude tapering [21,22].

To determine the ideal Tukey window for amplitude tapering of the 10-subwoofer array, a linear sweep of the tapering coefficient (0 – 1) was simulated (Fig. 4.12). The analysis reveals that the ideal amplitude tapering coefficient (with a Tukey window) is 0.30, resulting in an APR of 0.71.

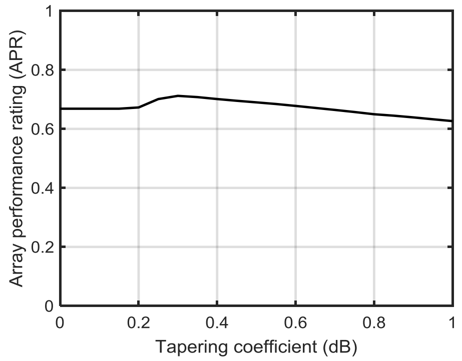

Fig. 4.12 APR after amplitude tapering applied over all possible points in the audience area

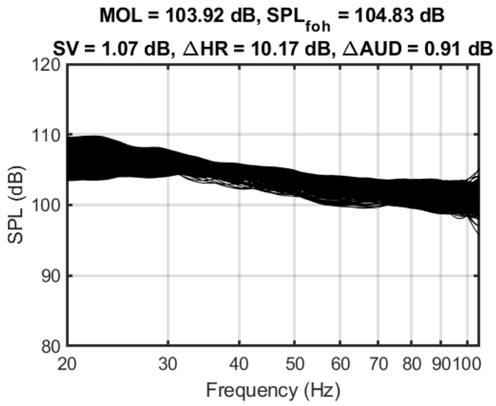

Fig. 4.13 Normalized magnitude responses for the 10-subwoofer system with subwoofer delay, position compensation and amplitude tapering applied

The amplitude tapering has served to slightly improve SV at the cost of ΔAUD. This is a compromise that must be considered on a case-by-case basis. In this particular scenario, amplitude tapering provides a slight improvement (0.4 dB) in system efficiency, therefore it will be used. This is contrary to the common belief that amplitude tapering will always lower system efficiency.

The overall improvement in performance can be judged by the normalized magnitude responses of the pre- and post-optimized systems (Figs. 4.6 and 4.13, respectively). This is seen as an improvement in APR from 0.57 to 0.71 (from a C to a B grade). The individual metrics are given in Table 4.2.

Metric	*Pre-optimization*	*Post-optimization*
APR	0.57	0.71
SV	2.03	1.07
ΔHR	7.80	10.17
ΔAUD	5.62	0.91

Table 4.2 Pre- and post-optimization metrics for the 10-subwoofer central system with 60 Hz spacing

The key improvement in system performance is reflected in the change in ΔAUD. A drop from 5.62 dB to 0.91 dB indicates that the mix position is receiving a more representative level in terms of audience experience. While system headroom (in reference to FOH SPL) has dropped by 2.37 dB, this is likely to be acceptable since the optimized system still supports 9.83 dB of headroom.

This reflects the importance of choosing the best possible system layout before optimization. The unoptimized array performs quite well on its own, but optimization further improves performance.

4.3 APR with non-ideal layouts

As a final example, consider a touring engineer arriving at a music festival. The system is a central array of only four subwoofers spaced at 50 Hz. The array can't be significantly repositioned, so optimization must be predominantly DSP-based.

Using the APR optimization process, the engineer inputs the configuration into software to determine the best subwoofer delay point. This results in an ideal delay point at (2.5 m, 25 m), bringing the APR from 0.32 to 0.46 (from an F to a D grade). Position compensation, as discussed in Section 4.2, can be applied which raises the APR to 0.52.

Finally, the engineer can determine if amplitude tapering is appropriate. The APR optimization indicates that a tapering coefficient of 0.70 will improve APR to 0.55 (C grade). All metrics covering this example system optimization are given in Table 4.3.

Metric	*(a)*	*(b)*	*(c)*	*(d)*
APR	0.32	0.46	0.48	0.55
SV	10.68	7.20	3.29	1.76
ΔHR	8.92	13.62	11.51	12.99
ΔAUD	10.11	2.85	4.15	4.23

Table 4.2 Optimization metrics (all expressed in dB other than APR) for the 4-subwoofer central system. (a) Original, (b) time-alignment, (c) time-alignment and position compensation, (d) time-alignment, position compensation and amplitude-tapering

In this scenario, the array layout was far from perfect. Optimization improved APR from an F to a C grade, though, resulting in better audience consistency and agreement between FOH and audience SPL. System headroom decreased, though, from 11.08 to 7.01 dB. In practice this process would be completely automated after the engineer inputs initial conditions, constraints and weightings.

5 Conclusions

A single metric (APR) quantifying live sound reinforcement subwoofer system performance has been proposed in order to provide engineers the ability to tailor systems to meet the needs of an event while working within the constraints of a venue/system. The optimization is designed to determine the ideal configuration automatically, using a set of given constraints. Such a metric would be of use within system design software, to help save time in optimization (avoiding much of the trail-and-error fine-tuning).

Further work is needed to refine the APR metric (and individual metric weightings) to ensure robustness and applicability to all reasonable scenarios. Additionally, temporal performance must be addressed since APR currently only operates on frequency domain data. With this accomplished, the metric would ideally be built into software to help system engineers optimize subwoofer arrays b the click of a button.

While APR brings nothing new in terms of specific optimization techniques, it provides engineers with the potential for optimization automation – something that would save precious time, both on- and off-site.

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